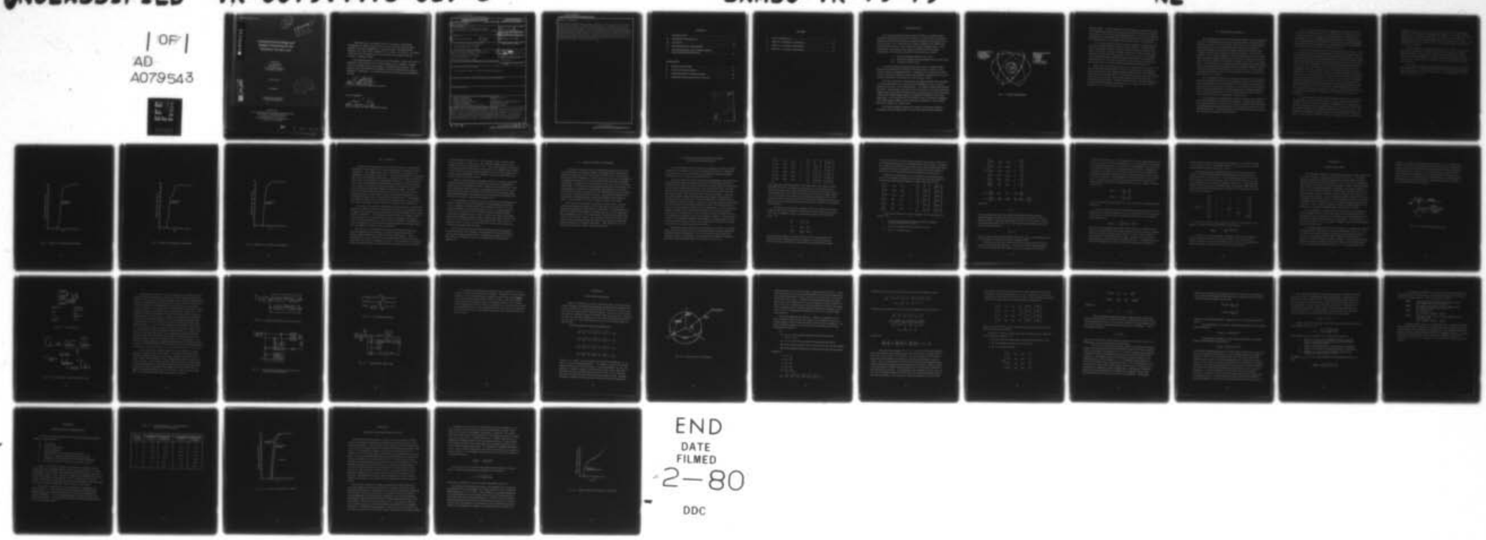
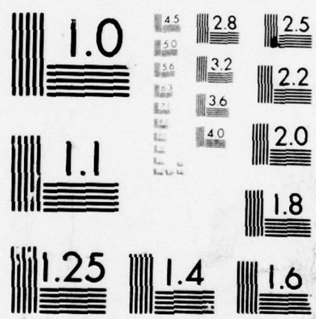
AD-A079 543 AEROSPACE CORP EL SEGUNDO CA F/G 17/7
COMBINED PSEUDO RANGE AND DOPPLER POSITIONING FOR THE STATIONAR--ETC(U)
MAR 79 P S JORGENSEN F04701-78-C-0079
UNCLASSIFIED TR-0079(4475-01)-3 SAMSO-TR-79-79 NL

| OF |
AD
A079543



END
DATE
FILMED
2-80
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

REPORT SAMSO-TR-79-79

AD A079543

(12) B.S.

LEVEL II

Combined Pseudo Range and Doppler Positioning for the Stationary Navstar User

Prepared by

P. S. JORGENSEN
Programs Group
The Aerospace Corporation
El Segundo, Calif. 90245

20 March 1979

Final Report

APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED

Prepared for

SPACE AND MISSILE SYSTEMS ORGANIZATION
AIR FORCE SYSTEMS COMMAND
Los Angeles Air Force Station
P.O. Box 92960, Worldway Postal Center
Los Angeles, Calif. 90009

80 1 17 006

This final report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract F04701-78-C-0079 with the Space and Missile Systems Organization, Deputy for Space Communications Systems, P.O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009. It was reviewed and approved for The Aerospace Corporation by E. M. Lassiter, Programs Group.

This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be available to the general public, including foreign nations. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

J. A. Strada

JOSEPH A. STRADA, LCDR, USN
Director of Navigation Equipment & Avionics

FOR THE COMMANDER

Joseph C. Carling

JOSEPH C. CARLING, Lt Col, USAF
Assistant Deputy for Space Navigation Systems

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 19
18 SAMSO-TR-79-79

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
6 Combined Pseudo Range and Doppler Positioning for the Stationary Navstar User.

5. TYPE OF REPORT & PERIOD COVERED
9 Final repts

6. PERFORMING ORG. REPORT NUMBER
14 TR-0079(4475-01)-3

7. AUTHOR(s)
10 Paul S. Jorgensen 12 46

8. CONTRACT OR GRANT NUMBER(s)
15 F04701-78-C-0079

9. PERFORMING ORGANIZATION NAME AND ADDRESS
The Aerospace Corporation
El Segundo, Calif. 90245
009 500

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS
Space and Missile Systems Organization
Air Force Systems Command
Los Angeles Air Force Station
P. O. Box 92960
Los Angeles, Calif. 90009

12. REPORT DATE
11 20 Mar [redacted] 79

13. NUMBER OF PAGES
42

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Biphase Modulation	Manpack Receiver
Doppler Positioning	Navstar
Global Positioning System	Pseudorandom Noise Modulation
Integrated Doppler	PRN Code Tracking
L-band Carrier Phase Lock Loop	Satellite Geodesy

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An early success in testing Navstar user equipment was achieved with the Manpack receiver. Using only two satellites, Navstars 1 and 2, the receiver accurately performed two-dimensional positioning while stationary. It was, in effect, combining pseudo range and integrated Doppler data. This concept of combining both types of data can be extended to three dimensions and to the operational Phase III Navstar. The stationary user can accomplish three- over

DD FORM 1473 (FACSIMILE)

UNCLASSIFIED 009 500

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

xlt

Abstract (Continued)

dimensional navigation with only three satellites. This capability is of interest to the Army since it would permit the stationary use of Navstar with much greater elevation mask angles. With the baseline 24-satellite constellation, three-dimensional navigation can be accomplished with a 30-deg mask angle, but it will take a few minutes longer because of the extra time required for making integrated Doppler measurements. Using this technique, three-dimensional position will be obtained to within 16 meters about 90 percent of the time. This operational capability will enhance the Army's ability to use Navstar in the presence of jamming, foliage, and terrain.

CONTENTS

FIGURES

I. INTRODUCTION

The Navstar/Global Positioning System (GPS) is a satellite-based navigation system that will provide extremely accurate three-dimensional position and velocity information to properly equipped users anywhere on or near the earth. It is a Joint Service Program managed by the USAF with deputies from the Navy, Army, Marines, Defense Mapping Agency, and Coast Guard, and with technical support provided by The Aerospace Corporation. The baseline program is divided into three phases:

I - Concept Validation Phase (1974-1979)
II - Full-Scale Engineering Development Phase (1979-1983)
III - Production Phase (1983-   )

The current baseline orbital configuration for the fully operational Phase III GPS employs 24 satellites in 55-deg inclined, circular, 12-hr orbits to transmit navigation signals. Continuous three-dimensional global coverage will be provided by placing eight satellites, equally spaced, in each of three orbit planes 120 deg apart in inertial space (Fig. 1).

The major elements comprising the navigation payload on the satellites are the pseudorandom noise signal assembly (PRNSA), atomic frequency standard, processor, and L-band antenna. The PRNSA includes the baseband generator, which produces the basic P (precise) and C/A (coarse/acquisition) ranging codes and encodes navigation data from the processor onto the pseudo-random noise (PRN) ranging signal; the amplifier/modulator units, which supply the L_1 (1575 MHz) and L_2 (1227 MHz) carrier frequencies modulated by the PRN ranging signals; and the high power amplifiers, which amplify the carrier signals for transmission.

The Control Segment consists of a Master Control Station (MCS), a navigation message Upload Station (ULS), and widely separated Monitor

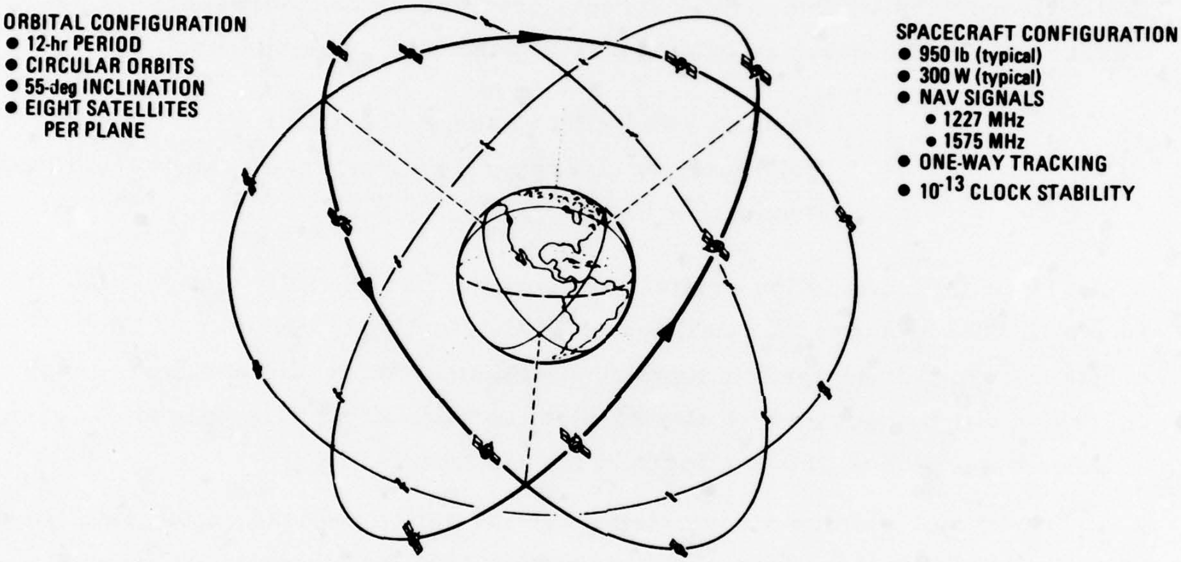


Fig. 1. Orbital Configuration

Stations (MSs). The MSs passively track all satellites in view and accumulate ranging data, which is processed at the MCS to calculate the satellite ephemerides, clock drifts, and propagation delay. At least once a day this information is transmitted by the ULS to the satellites for subsequent down-link transmission of the navigation data encoded on the carrier signals.

The conventional user measures pseudo range and pseudo range rate using the navigation signal from each of four satellites. "Pseudo range" is the true distance from the satellite to the user plus an offset due to the user's clock bias. Similarly, "pseudo range rate" is the true slant range rate plus an offset due to the frequency of the user's clock. Each signal carries ephemeris data and system timing information for that satellite. This allows the user receiver/processor to convert the pseudo range and pseudo range rate to user three-dimensional position and velocity.

Obviously, a stationary user has no need to determine his velocity, so only pseudo range data is needed. However, pseudo range rate (Doppler frequency offset) provides additional position information. The stationary user has the special opportunity to simultaneously process both types of data to determine his position, which allows position determination in three dimensions using only three satellites instead of four. To obtain sufficiently accurate integrated Doppler measurements, the stationary user must operate his receiver a few minutes longer.

II. DISCUSSION AND RESULTS

By combining pseudo range and integrated Doppler data, the stationary user has the advantage of two types of navigation data even though only one satellite is tracked. From the pseudo range data the user obtains position information just as the conventional Navstar user does, and from the integrated Doppler measurements he obtains position information via the technique geodetic users employ with Transit satellites. Each Navstar satellite serves as two, allowing for both the usual Navstar pseudo ranging and Transit-like Doppler positioning. This duality of operation is particularly effective because the "two satellites" appear to the user to be 90 deg apart. The pseudo range measurement provides position information along the line of sight (LOS) to the satellite, and the integrated Doppler measurement provides information perpendicular to this LOS (it provides position information along the direction of the component of relative velocity that is perpendicular to the LOS).

The conventional Navstar user requires pseudo range measurements from four satellites because he must compensate for the user clock phase bias, which constitutes a fourth unknown in the navigation computations (in addition to the three position components). The stationary user using both types of data is confronted with a fifth unknown in his navigation solution: the user clock frequency bias. For this reason three satellites must be used to obtain at lease five independent measurements.

The stationary user could perform pseudo ranging to three satellites and integrated Doppler measurements to only two. An alternate approach is to make both types of measurement to all three satellites and from these six independent measurements perform a least squares solution of the five unknown quantities. This latter approach is the method used in generating the quantitative results given in this report.

One interesting result of these computations is that whenever there are only three satellites visible to the stationary user (because of either a very high elevation mask angle or a small number of satellites in the constellation), the user always obtains a good solution. This differs from the conventional approach in which the user may not get a good navigation solution when only four satellites are available because the four satellites as a group have a poor geometry relative to the user.

The power of using both types of data arises from two factors. First, the navigation solutions (as simulated for this study) are obtained from an overdetermined set of equations (six equations with five unknowns). Second, the pseudo range and integrated Doppler data are inherently complementary. This can be illustrated by considering a seemingly poor geometric arrangement of only three satellites, all at rather high elevation angles relative to the user. Let us consider the case in which one satellite crosses the orbit plane of the other satellites in the constellation and these other two satellites are on opposite sides of the first (crossing) satellite: These three satellites are lined up in a row, so their geometry does indeed seem quite poor to a user directly beneath them. However, the velocity of the middle satellite is almost perpendicular to that of the other two satellites and is also perpendicular to the plane defined by the instantaneous position of all three. This direction of the velocity of the middle satellite is precisely the additional information needed by the stationary user to obtain a position component perpendicular to the plane common to himself and the three satellites.

The stationary user obtains a good solution as long as three satellites are available. This is illustrated by three cases: In the first case three satellites within the baseline 24-satellite constellation are visible above an elevation mask angle of 30 deg about 99.8 percent of the time, and the user does indeed obtain a good navigation solution 99.8 percent of the time. The cumulative probability of position dilution of precision (PDOP) is shown in Fig. 2. (The meaning of PDOP is discussed in detail later; for the present consider it to be an error ratio, the ratio of the user's three-dimensional position error to his error in ranging to the satellite.) The PDOP generally lies between 2.5 (10th percentile) and 3.1 (90th percentile). Since the ranging accuracy of the overall Navstar system is expected to be about 5.3 m, the stationary user will obtain his three-dimensional position to within 16 m 90 percent of the time.

In the second case, with only 12 satellites and a 5-deg elevation mask angle, three satellites are visible about 97 percent of the time, and once again the stationary user obtains a good navigation solution for all cases when at least three satellites are visible (Fig. 3 shows the PDOP statistics for this case). Again, the user obtains his position to within 16 m 90 percent of the time.

In the third case, with 15 satellites and a 10-deg elevation mask angle, three satellites are visible 99 percent of the time, and the user obtains his position to within 14 m 90 percent of the time (the PDOP statistics for this case are shown in Fig. 4).

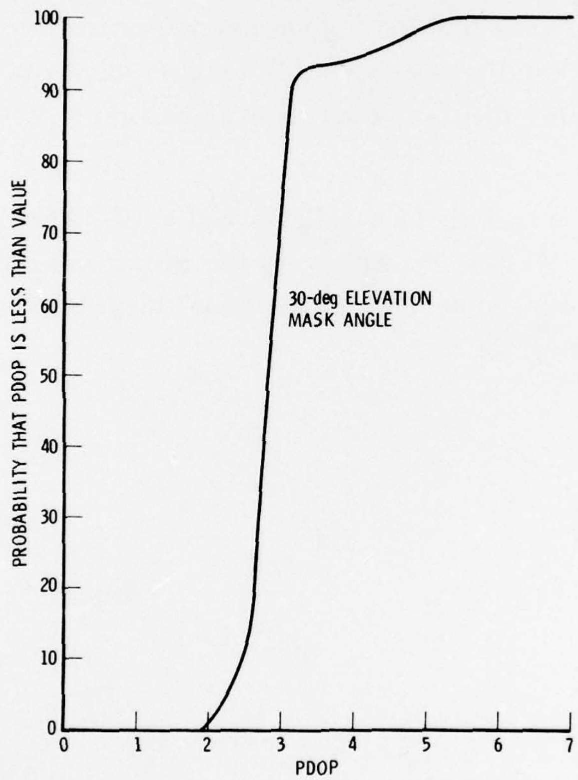


Fig 2. PDOP for 24-Satellite Constellation

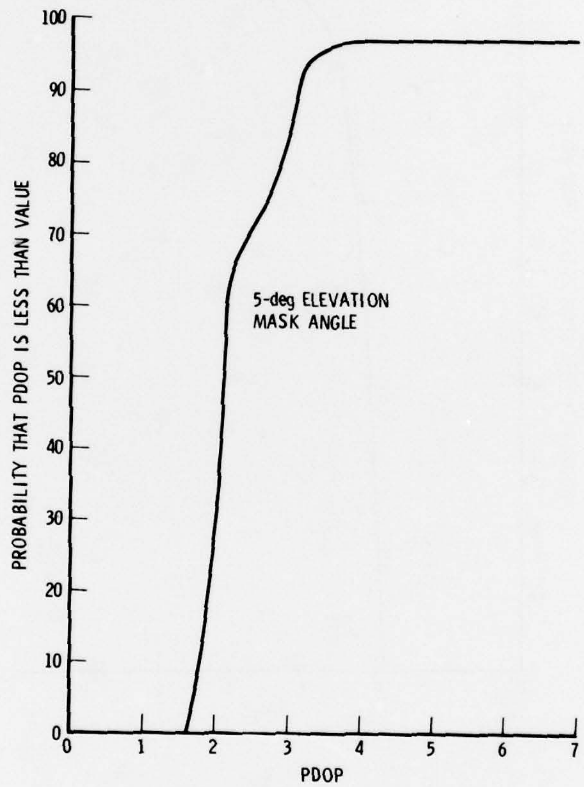


Fig. 3. PDOP for 12-Satellite Constellation

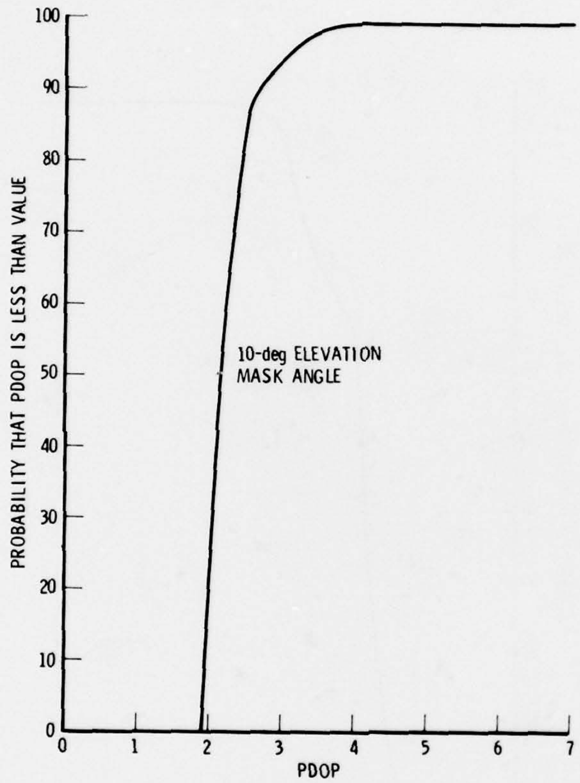


Fig. 4. PDOP for 15-Satellite Constellation

III. ACCURACY

Pseudo range measurements are made by tracking the PRN biphase modulation of the L-band carrier (see the discussion of signal structure in Appendix A). The chipping rate of this modulation is 10.23 MHz, corresponding to a wavelength of about 30 m. The integrated Doppler is measured with a phase lock loop that tracks the L_1 carrier whose frequency is 1575.42 MHz, which corresponds to a wavelength of about 19 cm. The granularity of these two tracking operations thus differs by about two orders of magnitude, and the accuracy of the integrated Doppler can therefore be expected to be much greater than that obtained with pseudo ranging. This high degree of integrated Doppler measurement accuracy is crucial to the stationary Navstar user who wishes to obtain an accurate position fix in a reasonable amount of time via both types of measurements. (Incidentally, this highly accurate carrier tracking is also important to current geodetic users, who now obtain accurate position fixes using Transit satellites.)

The analysis of satellite constellation performance with a conventional Navstar user (who uses pseudo range measurements to four satellites) uses a technique referred to as geometric dilution of precision (GDOP), which is discussed in Appendix B. The quantity generally of interest is PDOP, which is a dimensionless error ratio (the ratio of three-dimensional user position error to pseudo ranging error). The baseline (24-satellite) constellation is defined in Appendix C, which includes a graph showing the statistical cumulative probability of PDOP for this constellation.

For the case of a stationary user who uses both pseudo range and integrated Doppler measurements, it would be convenient to define a similar nondimensional error ratio to describe performance. This is complicated by the fact that we are now dealing with two types of measurements that may not be of comparable accuracy. In this report it was assumed that the integrated Doppler measurements are 50 times more accurate than the pseudo range measurements. (For example, if the accuracy (1σ) in pseudo ranging were about 5.3 m, the integrated Doppler measurement would be accurate to within about 11 cm. This level of Doppler performance is consistent with current Navstar test results.) The PDOP is thus defined as the ratio of three-dimensional navigation error to pseudo ranging error, with the integrated Doppler error considered to be one-fiftieth that of the pseudo range measurement errors.

Another consideration is the length of time over which the integrated Doppler measurement should be taken (about 2 min seems appropriate; during this time the satellites will have rotated in their orbits about 1 deg). It has been shown that using this 2-min assumption and the 50:1 accuracy ratio assumption leads to a solution in which the errors in pseudo range and integrated Doppler measurements each contribute about equally to the stationary user three-dimensional navigation error.

The quantitative results given in Sec. II are based on computations in which it is assumed that three satellites are simultaneously tracked for 2 min. (Stationary users generally have sequential receivers, so three integrated Doppler measurements require 6 min.) Two points about the "extra" time the stationary user needs to perform integrated Doppler measurements should be noted. First, in many cases only three pseudo range and two Doppler measurements are strictly necessary, which would increase the time by only 4 min. Second, during this "extra" time the stationary user's receiver is performing other necessary functions, e.g., reading the navigation data message.

Note that position could be obtained using only integrated Doppler measurements from four satellites; this parallels the conventional case of four pseudo range measurements. Although it is outside the scope of this report, this approach has been investigated and is discussed in Appendix D.

IV. COMPUTATIONAL TECHNIQUES

A computer program has been developed that divides the earth into areas bounded by increments of latitude and longitude and computes the GDOP parameters for a user at the center of these areas. The program determines which satellites are visible to the user and searches through all combinations to find the set of three satellites that yields the best GDOP parameters. In general, the criterion most often used is the three-dimensional PDOP. To take into account satellite motion and earth rotation, the computations are repeated for all latitudes and longitudes at successive time increments; thus, the statistics of the GDOP parameters are obtained for any time of day and any place on earth. In general the statistics are computed on a worldwide basis, giving equal weight to all locations on earth. These statistics are used to obtain cumulative percentiles.

For each of the statistical results given in this report, a total of 3,240 computations is performed. For each of these, the best three satellites are obtained, and the PDOP is computed. The Northern Hemisphere is divided into 18 latitudes, separated by 5 deg, and 12 longitudes, also separated by 5 deg. The computations, which are performed for the center of each of these "boxes," are repeated 15 times over a time span during which the satellites rotate through an angle equal to the spacing between satellites in a common orbit plane. This effectively covers all locations on the earth's surface and all times of day.

V. EQUATIONS USING BOTH PSEUDO RANGE AND INTEGRATED DOPPLER

The conventional Navstar technique of navigating by means of pseudo ranging to four satellites is discussed in Appendices B and C. Equations that apply to the case in which both pseudo range and integrated Doppler measurements are employed are presented here.

Making pseudo range measurements at the beginning of a time interval and performing integrated Doppler measurements during this interval is mathematically equivalent to making two sets of pseudo range measurements, one set at the beginning and the other at the end of the interval. The integrated Doppler measurements are the same as the difference between the two sets of pseudo range measurements to the same satellite. (In theory, one could imagine an approach using only pseudo range measurements, enabling the user to employ fewer than four satellites. The difficulty with this approach lies in the random errors from the pseudo range measurements; for this approach to work with reasonable accuracy, the time between the two sets of measurements would have to be about an hour.) Using integrated Doppler measurements, which take advantage of the very high accuracy with which the L-band carrier can be tracked, the user can obtain an accurate position determination in just a few minutes. However, from the point of view of developing the equations associated with combining the two types of measurements, this process can be considered the mathematical equivalent of two sets of pseudo range measurements.

Linearized solution equations for obtaining user position from pseudo range measurements only have been developed and conveniently expressed in matrix notation (Appendix B). The case of two time points and three satellites involves six equations in five unknowns that relate user position and clock offsets to pseudo range measurements are given as

$$
\begin{bmatrix}
\alpha_{111} & \alpha_{121} & \alpha_{113} & 1 & 0 \\
\alpha_{211} & \alpha_{221} & \alpha_{231} & 1 & 0 \\
\alpha_{311} & \alpha_{321} & \alpha_{331} & 1 & 0 \\
\alpha_{112} & \alpha_{122} & \alpha_{132} & 0 & 1 \\
\alpha_{212} & \alpha_{222} & \alpha_{232} & 0 & 1 \\
\alpha_{312} & \alpha_{322} & \alpha_{332} & 0 & 1
\end{bmatrix}
\times
\begin{bmatrix}
\Delta_x \\ \Delta_y \\ \Delta_z \\ \Delta T_1 \\ \Delta T_2
\end{bmatrix}
=
\begin{bmatrix}
\Delta R_{11} \\ \Delta R_{21} \\ \Delta R_{31} \\ \Delta R_{12} \\ \Delta R_{22} \\ \Delta R_{32}
\end{bmatrix}
$$

where α_{ijk} is the direction cosine of the angle between the range to the ith satellite and the jth coordinate at the kth time; Δx, Δy, and Δz are the differences between the actual and nominal (a priori best estimate) values of user position; ΔT_k is the difference between the actual and nominal values of user clock bias at the kth time (ΔT_1 and ΔT_2 will differ because of the offset of the user's clock frequency from nominal); and ΔR_{ik} is the difference between the actual and nominal pseudo range measurement to the ith satellite at the kth time.

The last three pseudo range equations at the second time point can be replaced with integrated Doppler equations by taking the difference between each of these equations and the corresponding equation at the first time point. Let

$$
\begin{aligned}
\beta_{ij} &= \alpha_{ij2} - \alpha_{ij1} \\
\Delta t &= \Delta T_2 - \Delta T_1 \\
\Delta D_i &= \Delta R_{i2} - \Delta R_{i1}
\end{aligned}
$$

where β_{ij} is the change in the direction cosine of the angle between the range to the ith satellite and the jth coordinate over the time interval, Δt is the change in the difference between the actual and nominal values of user clock bias over the time interval (the average offset of the user's clock frequency from nominal multiplied by the time interval), and ΔD_i is the change in the difference between the actual and nominal pseudo range measurement to the ith satellite over the time interval (the difference between the actual and nominal integrated Doppler measurements).

With these substitutions, the six equations in five unknowns that relate user position, clock bias, and average clock frequency offset to pseudo range and integrated Doppler measurements are given by

$$\begin{bmatrix} \alpha_{111} & \alpha_{121} & \alpha_{131} & 1 & 0 \\ \alpha_{211} & \alpha_{221} & \alpha_{231} & 1 & 0 \\ \alpha_{311} & \alpha_{321} & \alpha_{331} & 1 & 0 \\ \beta_{11} & \beta_{12} & \beta_{13} & 0 & 1 \\ \beta_{21} & \beta_{22} & \beta_{23} & 0 & 1 \\ \beta_{31} & \beta_{32} & \beta_{33} & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta T_1 \\ \Delta t \end{bmatrix} = \begin{bmatrix} \Delta R_{11} \\ \Delta R_{21} \\ \Delta R_{31} \\ \Delta D_1 \\ \Delta D_2 \\ \Delta D_3 \end{bmatrix}$$

Using matrix notation, the above equations can be expressed compactly. Let

r = the six-element pseudo range and integrated Doppler measurement difference vector

x = the user position and clock correction vector

A = the 6×5 solution matrix

$$A \equiv \begin{bmatrix} \alpha_{111} & \alpha_{121} & \alpha_{131} & 1 & 0 \\ \alpha_{211} & \alpha_{221} & \alpha_{231} & 1 & 0 \\ \alpha_{311} & \alpha_{321} & \alpha_{331} & 1 & 0 \\ \beta_{11} & \beta_{12} & \beta_{13} & 0 & 1 \\ \beta_{21} & \beta_{22} & \beta_{23} & 0 & 1 \\ \beta_{31} & \beta_{32} & \beta_{33} & 0 & 1 \end{bmatrix}$$

$$x \equiv \begin{bmatrix} \Delta x & \Delta y & \Delta z & \Delta T_1 & \Delta t \end{bmatrix}$$

$$r \equiv \begin{bmatrix} \Delta R_{11} & \Delta R_{21} & \Delta R_{31} & \Delta D_1 & \Delta D_2 & \Delta D_3 \end{bmatrix}$$

Therefore

$$Ax = r$$

which compactly expresses the relationship between pseudo range and integrated Doppler measurements and user position, clock bias, and clock frequency offset. Since this relationship is linear, it can be used to express the relationship between the errors in the measurements and the user quantities and can be stated as

$$A\epsilon_x = \epsilon_r$$

where ϵ_r represents the measurement errors and ϵ_x the corresponding errors in user position, clock bias, and clock frequency offset.

Let us now consider the covariance matrices of the expected measurement errors and the user quantities. The first covariance measurement is a 6×6 array composed of the expected values of the squares and products of the measurement errors. The diagonal terms in the matrix (the squares of the expected errors) are the variances (i.e., the squares of the expected one sigma measurement error values). The off-diagonal terms represent the covariance between the measurements and reflect the correlations to be expected. Similarly, the covariance matrix for the user quantities is composed of the expected values of the squares and products of the errors in the user quantities. The diagonal terms are the variance or the squares of the errors in user position and clock parameters (1σ), while the off-diagonal terms reflect the correlations in these errors. These covariance matrices are

$$COV(r) = E\left\{\epsilon_r \epsilon_r^T\right\}$$

$$COV(x) = E\left\{\epsilon_x \epsilon_x^T\right\}$$

where the symbol $E\{\ \}$ designates the expected value of the quantity inside the braces.

When a weighted least squares solution is performed for the five user quantities from the six measurements, the relationship between the measurement covariance matrix and the covariance matrix for the user quantities can be shown to be

$$COV(x) = \left[A^T COV(r)^{-1} A\right]^{-1}$$

Note that the relationship between the measurement errors and the user's errors is a function only of the solution matrix A, which in turn is a function only of the direction cosines of the LOSs from the user to the satellites along whichever coordinate system is used. In other words, the error relationships are functions only of satellite geometry. An important consideration in the proper use of Navstar is the fact that the three satellites being used must possess "good" geometric properties. (A "good" situation is one in which, because of satellite geometry, a given level of error in the measurements results in small user errors.)

Certain assumptions regarding pseudo range measurement errors lead to a method of quantitatively determining whether a particular three-satellite geometry is good or bad. Let each individual pseudo range measurement have an error (1σ) of unity, where the expected mean is zero and the correlation of errors between satellites is also zero. If the integrated Doppler measurements are one-fiftieth those of the pseudo range measurements (as discussed previously), the reciprocal of the measurement covariance matrix is given by

$$COV(r)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2500 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2500 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2500 \end{bmatrix}$$

Let Vx, Vy, and Vz be the variances of the three components of user position. The three-dimensional PDOP is then given by

$$PDOP = \sqrt{Vx + Vy + Vz}$$

The "best" set of three satellites is that set, out of all combinations visible to the user, that has the minimum PDOP. The cumulative percentiles of PDOP for various combinations of total number of satellites and assumed elevation mask angle are shown in Figs. 2 through 4.

APPENDIX A

SIGNAL STRUCTURE

Each satellite transmits a navigation signal on two L-band frequencies, one at 1575.42 MHz (L_1) and the other at 1227.6 MHz (L_2). These two carrier frequencies are biphase modulated by pseudo random sequences providing a spread spectrum modulation. The L_1 carrier is actually modulated by two such sequences in phase quadrature so that, strictly speaking, this carrier is actually quadraphase modulated. One pseudo random sequence is a precision (P) signal at a random pulse repetition rate of 10.23 MHz and is an extremely long code so that for all practical purposes it is a truly random sequence. The second pseudo random sequence is a coarse/acquisition (C/A) signal, which is a short sequence used either for initial acquisition of the P signal or as a less accurate navigation signal for low-cost users. The L_2 carrier frequency is biphase modulated only by the P signal or, as a ground-controlled option, only by the C/A signal.

For the purpose of this discussion we will confine our attention only to the P signal on the L_1 carrier; this is the primary navigation signal. Note that the carrier frequency (1575.42 MHz) is an exact multiple (154) of the pseudo random sequence pulse rate (10.23 MHz). The wavelength of the carrier is only 19 cm, whereas the "chipping" rate of the pseudo random sequence is about 30 m.

The pseudo random sequence is generated by a feedback shift register, the output of which modulates the carrier as illustrated in Fig. A-1. The P signal pseudo random sequence generator is functionally illustrated in Fig. A-2. By combining four 12-stage feedback shift registers, the equivalent of a 48-stage shift register is obtained. Using a pseudo random sequence to biphase modulate the carrier results in the transmitted spectrum being spread as illustrated in Fig. A-3. This spread permits an interference signal from being rejected in the receiver in the following manner: The receiver's pseudo random sequence generator modulates the incoming signal in the same manner as the generator on the satellite (the two generators are the same). The original carrier frequency is thus reconstructed and is collapsed to a very narrow band. The interference signal, however, is spread out over a wide spectrum, and this signal can therefore be filtered out so that only a small residue remains near the now-reconstructed carrier frequency.

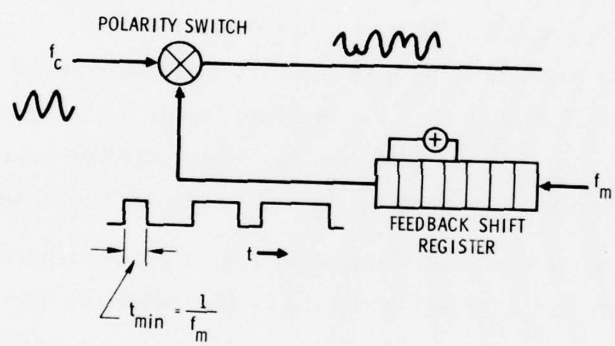
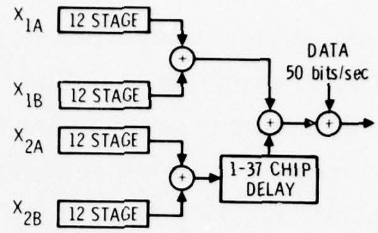


Fig. A-1. Pseudo Random (Noise) Code



CHARACTERISTICS	CODE PROPERTIES
RATE	10.23×10^6 bits/sec
PERIOD	$\approx 2^{48} - 1$ 267 days
TYPE CODE	LINEAR, NON-MAX LENGTH
CORRELATION PROPERTIES	~ 36 dB

Fig. A-2. Code Generator

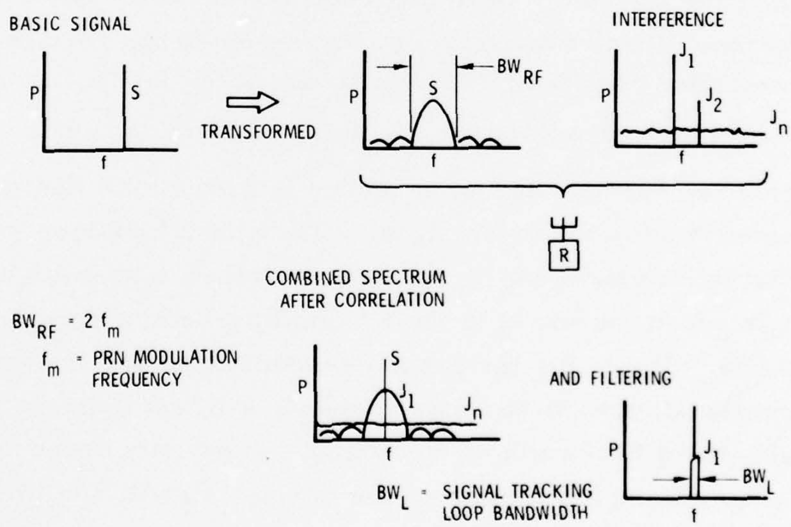


Fig. A-3. Pseudo Random Noise-Spread Spectrum

Functionally, Navstar receivers incorporate two tracking loops that must operate simultaneously to properly track the Navstar navigational signal. The first is the code tracking loop, which tracks the pseudo random sequence by matching the locally generated sequence with the sequence on the received signal. Simultaneously, a phase lock loop is tracking the carrier frequency. The actual process is much more complicated because of the necessary intermediate frequency (IF) downconversion steps. For simplicity, however, these downconversion steps are omitted in the functional diagram shown in Fig. A-4. Figure A-5 expands on the function of such a receiver by illustrating the role of a feedback shift register and envelope detectors in the code lock loop that tracks the incoming pseudo random sequence. There are three outputs of the feedback shift register: an on-time sequence P_O, an early sequence P_E, and a late sequence P_L. The early and late codes modulate the carrier frequency C, which is synthesized in the phase lock loop. These signals are then mixed with the incoming signal, thereby generating voltages proportional to the extent that the sequences match the incoming sequence from the satellite. The difference between these two signals generates an error voltage that drives a voltage-controlled oscillator (VCO) with which the feedback shift register is synchronized, thereby tracking the incoming random sequence (Fig. A-6).

The on-time pseudo random sequence is mixed with the incoming signal to reconstruct the carrier signal. The phase lock loop (which is actually a bistable Costas loop) tracks this satellite-transmitted carrier. Binary data is added modulo-2 to the P signal pseudo random sequence at a rate of 50 bps. Since only the pseudo random sequence is removed from the incoming signal, the 50-bps data sequence still remains on the signal, but the single-sided bandwidth of this signal is now only about 50 Hz. In addition to maintaining phase lock on the carrier signal, the Costas loop (Fig. A-7) also strips off the data remaining on the signal.

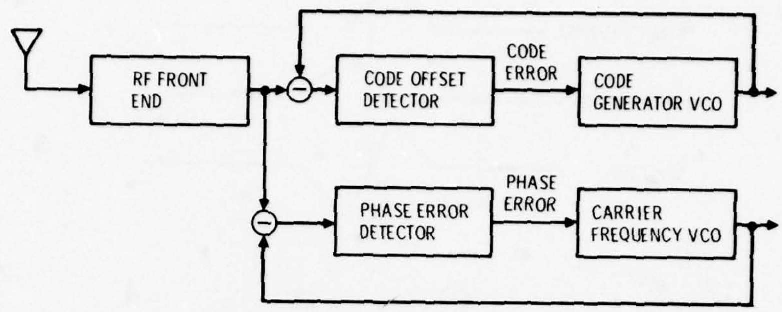


Fig. A-4. Simplified Diagram of Generic Receiver

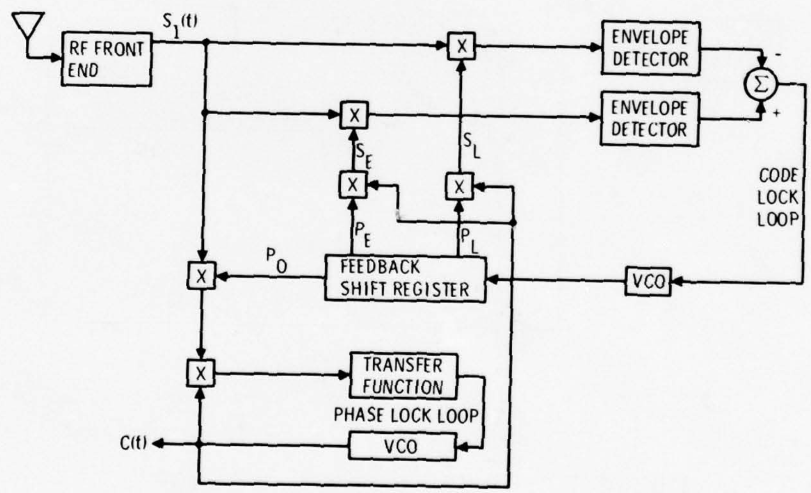


Fig. A-5. Generic Pseudo Random Noise Receiver Functional Block Diagram

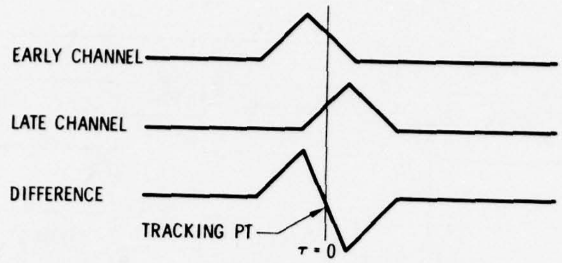


Fig. A-6. Correlation Principle

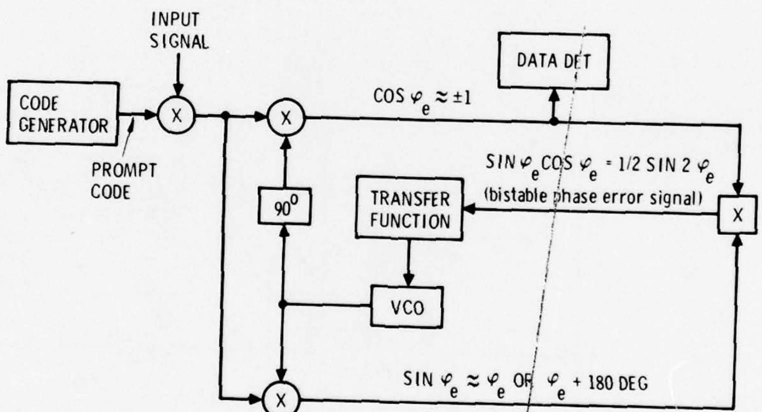


Fig. A-7. Costas (Phase Lock) Loop

Tracking the incoming pseudo random sequence allows the receiver to make a range measurement to the satellite. Tracking the carrier signal, however, does not provide absolute ranging measurements; only <u>changes</u> in the range can be measured from carrier tracking. Since the chipping rate of the pseudo random sequence corresponds to about 30 m and since the wavelength of the carrier frequency is about 19 cm, there is a two-order-of-magnitude difference in the quantization of the code and carrier tracking loops, and the accuracy of the phase lock loop is much greater than that of the code loop.

APPENDIX B

NAVIGATION EQUATIONS

Figure B-1 illustrates an earth-centered inertial coordinate system. At zero time the x-axis passes through the intersection of the equator and prime meridian, the z-axis passes through the North Pole, and the y-axis completes the right-handed coordinate system. Because of earth rotation, the x and y coordinates change in longitude about 15 deg per hour. Shown on the figure are the user position (x, y, and z) and the position of satellite 1 (x_1, y_1, and z_1). The range distance between the user and satellite 1 is shown as R_1.

The basic equations using four satellites are

$$\sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} + T = R_1$$

$$\sqrt{(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2} + T = R_2$$

$$\sqrt{(x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2} + T = R_3$$

$$\sqrt{(x - x_4)^2 + (y - y_4)^2 + (z - z_4)^2} + T = R_4$$

where x, y, z, and T are user position and clock bias (unknowns), x_i, y_i, and z_i are the ith satellite position, i = 1, 4 (known), and R_i is the pseudo range measurement to the ith satellite. Here the quantities, R_1, R_2, R_3, and R_4 are "pseudo" ranges in that they are the sum of the actual range displacements plus the offset due to user time error. For convenience, units have been selected such that the velocity of light is unity. Roughly speaking, if displacement is measured in feet, then time is measured in

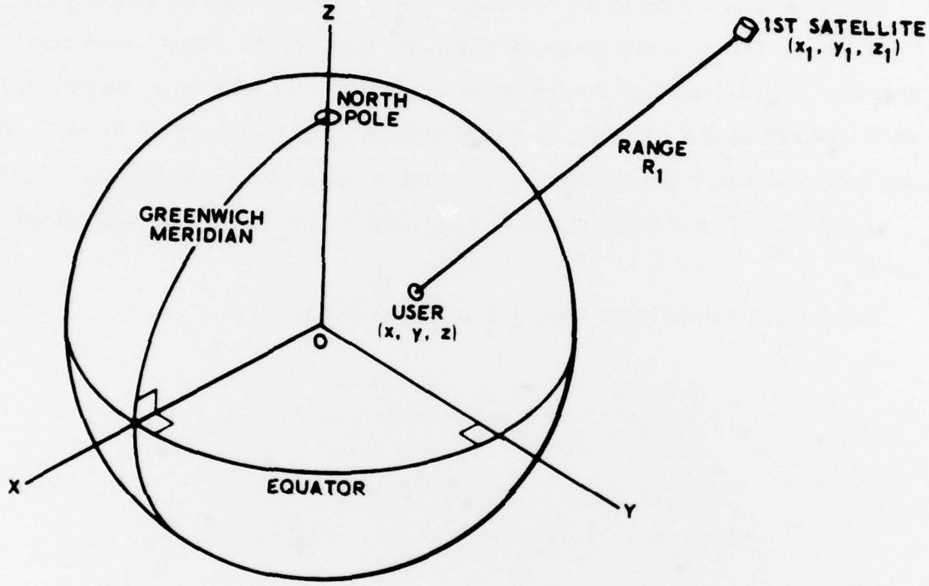


Fig. B-1. Earth-Centered Coordinates

nanoseconds since the velocity of light is approximately 10^9 ft/sec. In the equations shown here the four pseudo ranges are the measured quantities. The satellite positions are known, and the four unknowns are user position and the user clock error. It should be emphasized that while precision atomic frequency standards are used in the satellites and the monitor stations, there is no requirement for Navstar/GPS users to have a precision clock. Ordinary quartz crystal frequency standards are adequate for the user since he is continuously computing time for the four pseudo range measurements.

The above equations are nonlinear. While it is possible to solve these equations directly as they are shown, user equipments without exception employ a much simpler linearized version of these equations. The basic navigation equations can be linearized by employing incremental relationships as follows.

Let

x_n, y_n, z_n, T_n be nominal (a priori best estimate) values of x, y, z, T

Δx, Δy, Δz, ΔT be the corrections to these nominal values

R_{ni} be the nominal pseudo range measurements from the ith satellite

ΔR_i be the difference between the actual and nominal measurements

Therefore

$$x = x_n + \Delta x$$

$$y = y_n + \Delta y$$

$$z = z_n + \Delta z$$

$$T = T_n + \Delta T$$

$$R_i = R_{ni} + \Delta R_i$$

$$R_{ni} = \sqrt{(x_n - x_i)^2 + (y_n - y_i)^2 + (z_n - z_i)^2} + T_n$$

Substituting the incremental expressions into the basic equations yields

$$\sqrt{(x_n + \Delta x - x_i)^2 + (y_n + \Delta y - y_i)^2 + (z_n + \Delta_z - z_i)^2}$$
$$= R_{ni} + \Delta R_i - T_n - \Delta T, \; i = 1, 4$$

By ignoring second-order error terms, these equations can be written as

$$\sqrt{(x_n - x_i)^2 + (y_n - y_i)^2 + (z_n - z_i)^2}$$
$$+ \frac{(x_n - x_i)\Delta x + (y_n - y_i)\Delta y + (z_n - z_i)\Delta z}{\sqrt{(x_n - x_i)^2 + (y_n - y_i)^2 + (z_n - z_i)^2}}$$
$$= R_{ni} + \Delta R_i - T_n - \Delta T$$

Substituting

$$\frac{(x_n - x_i)}{R_{ni} - T_n} \Delta x + \frac{(y_n - y_i)}{R_{ni} - T_n} \Delta y + \frac{(z_n - z_i)}{R_{ni} - T_n} \Delta z + \Delta T = \Delta R_i$$

The above four equations (i = 1, 2, 3, 4) are the linearized equations that relate pseudo range measurements to the desired user navigation information as well as the user's clock bias. The known quantities of the right-hand side of the equation are actually incremental pseudo range measurements. They are the differences between the actual measured pseudo ranges and the measurements that had been predicted by the user's computer based on the knowledge of satellite position and the user's most current estimate of his position and clock bias. The quantities to be computed, Δx, Δy, Δz, and ΔT, are corrections that the user will make to his current estimate of position and clock biases. The coefficients of these quantities on the left-hand side are the direction cosines of the line of sight (LOS) from the user to the satellite as projected along the x, y, and z coordinates. For all four equations the coefficient in front of ΔT is unity. These linearized equations can be conveniently expressed in matrix notation and appear as

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & 1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & 1 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & 1 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & 1 \end{bmatrix} \times \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta T \end{bmatrix} = \begin{bmatrix} \Delta R_1 \\ \Delta R_2 \\ \Delta R_3 \\ \Delta R_4 \end{bmatrix}$$

where α_{ij} is the direction cosine of the angle between the range to the ith satellite and the jth coordinate.

By the use of matrix notation, the above equations can be expressed very compactly as follows. Let

r = the four-element pseudo range measurement difference vector

x = the user position and time correction vector

A = the 4×4 solution matrix

$$A \equiv \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & 1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & 1 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & 1 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & 1 \end{bmatrix}$$

$$x \equiv [\Delta x \quad \Delta y \quad \Delta z \quad \Delta T]^T$$

$$r \equiv [\Delta R_1 \quad \Delta R_2 \quad \Delta R_3 \quad \Delta R_4]^T$$

Therefore

$$Ax = r \qquad \text{or} \qquad x = A^{-1}r$$

The last equation presented compactly expresses the relationship between pseudo range measurements and user position and clock bias. Since this relationship is linear, it can be used to express the relationship between the errors in pseudo range measurement and the user quantities. This relationship is therefore

$$\epsilon_x = A^{-1}\epsilon_r$$

where ϵ_r represents the pseudo range measurement errors and ϵ_x, the corresponding errors in user position and clock bias.

Let us now consider the covariance matrix of the expected errors in pseudo range measurements and the covariance matrix of the user quantities. The first covariance measurement is a 4×4 array composed of the expected values of the squares and products of the errors in the pseudo range measurements. The diagonal terms in the matrix, namely the squares of the expected errors, are the variances, i.e., the squares of the expected $1\ \sigma$ values of the pseudo range measurement errors. The off-diagonal terms are the covariance between the pseudo range measurements and reflect the correlations to be expected in these measurements. Likewise the covariance matrix for the user quantities is composed of the expected values of the squares and products of the errors in the user quantities. The diagonal terms are the variance or the squares of the $1\,\sigma$ errors in user position and time, while the off-diagonal terms reflect the correlations in these errors. These covariance matrices are given by

$$COV(r) = E\left\{\epsilon_r \epsilon_r^{\,T}\right\}$$

$$COV(x) = E\left\{\epsilon_x \epsilon_x^{\,T}\right\}$$

where the symbol $E\{\ \}$ designates "expected value" of the quantity inside the braces.

Upon substitution, the matrix relationship between the two covariance matrices becomes

$$COV(x) = A^{-1} COV(r) A^{-T}$$

An alternate formulation for this relationship based on a straightforward matrix algebra manipulation is

$$COV(x) = [A^T COV(r)^{-1} A]^{-1}$$

Note that the relationship between the pseudo range measurement errors and the user's position and clock bias errors is a function only of the solution matrix A, which in turn is a function only of the direction cosines of the LOSs from the user to the satellites along whichever coordinate system is used. In other words, the error relationships are functions only of satellite geometry. An important consideration in the proper use of Navstar/GPS is the fact that the four satellites being used must possess "good" geometric properties. (A "good" situation is one in which, because of satellite geometry, a given level of error in the pseudo range measurements results in small user errors.) This leads to the concept of geometric dilution of precision (GDOP), a measure of how satellite geometry degrades accuracy.

The following assumption regarding pseudo range measurement errors provides a method of quantitatively determining whether a particular four-satellite geometry is good or bad. Let each individual pseudo range measurement have an error (1σ) of unity, where the expected mean is zero and the correlation of errors between satellites is also zero. With these assumptions the covariance matrix for the errors in the pseudo range measurements becomes a 4×4 unity matrix. Thus, for this case, the covariance matrix for user position and clock bias errors is given by

$$COV(x) = (A^T A)^{-1}$$

GDOP is defined as the square root of the trace of COV(x) when COV(r) is an identity matrix; therefore

$$GDOP = \sqrt{TRACE\left[(A^T A)^{-1}\right]}$$

Some properties of this quantity can be summarized as follows:

a. GDOP is, in effect, the amplification factor of pseudo range measurement errors into user errors due to the effect of satellite geometry.

b. GDOP is independent of the coordinate system employed.

c. GDOP is a criterion for designing satellite constellations.

d. GDOP is a means for user selection of the four best satellites from those that are visible.

By letting V_x, V_y, V_z, V_T be the variances of user position and time, we have

$$GDOP = \sqrt{V_x + V_y + V_z + V_T}$$

As an alternative to GDOP as a criterion for selecting satellites or evaluating satellite constellations, only some of the variances of user position and time might be used. These are defined as follows:

PDOP	The square root of the sum of the squares of the three components of position error
HDOP	The square root of the sum of the squares of the horizontal components of position error
VDOP	The altitude error Note: $PDOP^2 = HDOP^2 + VDOP^2$
TDOP	The error in the user clock bias multiplied by the velocity of light Note: $GDOP^2 = PDOP^2 + TDOP^2$

The alternative criterion most frequently used is the position dilution of precision (PDOP). PDOP is also invariant with the coordinate system and is used because the most important consideration in any navigation system is position accuracy; knowing time is generally a secondary by-product. Another alternative is the horizontal dilution of precision (HDOP), which is most meaningful for users who are using the system primarily to obtain horizontal position.

APPENDIX C

BASELINE ORBIT CONFIGURATION

The space segment configuration for the Phase III fully operational system is summarized as:

a. 24 satellites
b. 12-hr circular orbits
c. 55-deg inclination
d. Eight satellites in each of three orbit planes
e. Satellites in a plane uniformly spaced 45 deg apart
f. Orbit planes uniformly spaced 120 deg apart in longitude
g. Satellites staggered 15 deg from one plane to another

Table C-1 shows the statistics on the number of satellites that are visible under the assumption that they can be seen above either 5 or 10 deg in elevation. These results are the average for all locations on earth and for all times during the day. In general, eight or nine satellites can be seen at a given time above 5 deg, and six, seven, or eight can be seen above 10 deg. Since only four satellites are needed for navigation, users generally have twice as many available from which to select the best four.

The geometric performance of the baseline orbit configuration is shown in Fig. C-1. This figure shows the cumulative probability of the position dilution of precision (PDOP) for three-dimensional navigation using four satellites. A 55-deg inclination angle has been selected to optimize performance. The orbit planes intersect each other at about 90 deg and form eight equal octants.

Table C-1. Navstar Phase III: 3 × 8 Distribution of Global Satellite Visibility

No. of Satellites Visible	5-deg Elevation Mask Angle		10-deg Elevation Mask Angle	
	Probability, percent	Cumulative Probability	Probability, percent	Cumulative Probability
10	4.5	4.5	0	0
9	38.5	43.0	8.0	8.0
8	34.4	77.4	37.0	45.0
7	9.6	87.0	23.6	68.6
6	13.0	100	27.4	96.0
5	0	100	3.8	99.8
4	0	100	0.2	100

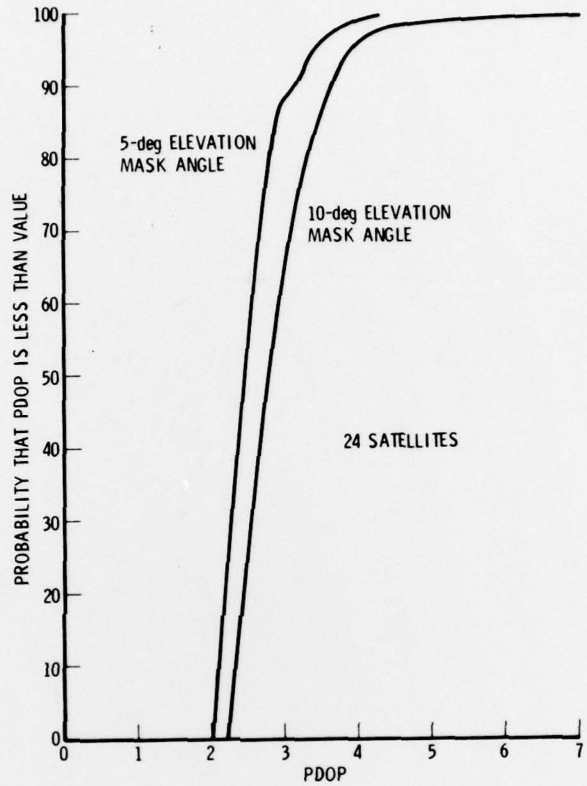


Fig. C-1. Cumulative Probability of PDOP

APPENDIX D

DOPPLER NAVIGATION WITH NAVSTAR

Navstar/GPS performs highly accurate, three-dimensional, instantaneous, and continuous navigation by obtaining pseudo range measurements from four satellites. Theoretically, pseudo range rate measurements could be used instead of pseudo range, and this idea (using integrated pseudo range rate, i.e., measuring the Doppler effect on the transmission from the satellite to the user via the fully operational Phase III GPS) has been investigated. The user simultaneously makes an integrated Doppler measurement from four satellites over an interval of time T. (For the purpose of this analysis it is assumed that the time intervals of the measurements from the satellites are all the same and that they are performed simultaneously.) The user has then acquired four measurements of range change over the interval T, and from these measurements he can compute the three components of position and the frequency offset of the local oscillator from nominal. Obtaining this frequency offset is analogous to the user's determining his clock bias in the normal case of making pseudo range measurements.

For the above approach to Doppler navigation the position dilution of precision (PDOP) can be defined as follows: Consider that the four range change measurements are made with an error (1σ) of unity and that these four measurement errors are uncorrelated. For these errors there will be a corresponding three-dimensional user navigation error (1σ) in computing the position: PDOP is defined as this three-dimensional position error and is the ratio of user error to measurement error. This ratio is a function of the relative geometry of the four satellites selected to perform Doppler navigation. To obtain the best possible accuracy, the user would select the set of four satellites, out of all those visible, that provide the minimum PDOP.

Using the above criteria for selecting the best four satellites, a computer program has been used to determine statistically the PDOP values that can be expected when Doppler navigation is employed. The results show that for all measurement intervals (T) up to about one hour, the PDOP varies inversely with T. This being the case, a PDOP coefficient can be defined by multiplying PDOP by the measurement interval. In this manner the statistical results can be condensed and are readily presented for the case of all measurement intervals. The cumulative probability distribution of K_1, which is PDOP multiplied by the measurement interval in seconds, is shown in Fig. D-1. The PDOP coefficient (1σ) is 1.27×10^4, and PDOP (1σ) is therefore given by

$$\mathrm{PDOP} \approx \frac{1.27 \times 10^4}{T}$$

Let σ_m be the error (1σ) in the integrated pseudo range rate measurement. The three-dimensional user position error is given by

$$E = \frac{1.27 \times 10^4 \times \sigma_m}{T}$$

Note that σ_m divided by T is the average pseudo range rate error.

To summarize the statistical results, user position error is directly proportional to the pseudo range rate measurement error; the constant of proportionality is 1.27×10^4 sec. For example, a completely stationary user could obtain an accurate position fix in 2 min if he measured the integrated pseudo range rate by tracking the 1.575-GHz carrier frequency. Assuming that this measurement is made with an accuracy of 11 cm over 120 sec, this corresponds to a measurement error of about 0.09 cm/sec, and the corresponding user position error will therefore be about 12 m.

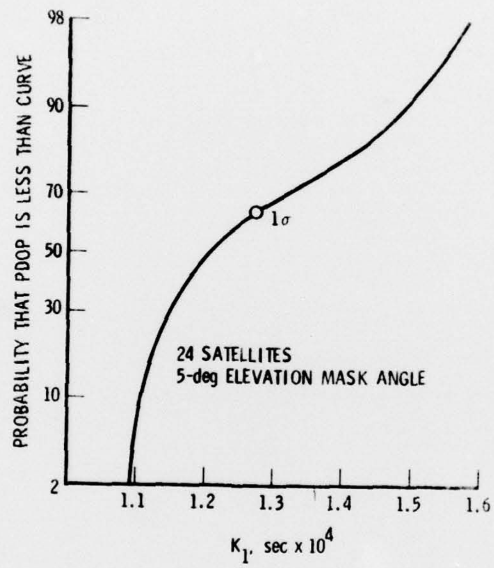


Fig. D-1. PDOP Coefficient for Doppler Navigation